

Topological Dyons

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Abstract

Using a two potential approach, dyon solutions have been found in the temporal and non-temporal gauges for a non-Abelian theory. Both the charges, electric and magnetic, of the temporal dyon solution are topological, while for the non-temporal case both charges are partially topological.

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I. INTRODUCTION

The works of 't Hooft [1] and Polyakov [2] demonstrated that a non-Abelian gauge field theory coupled to a self-interacting scalar field gives rise to magnetic monopole. This magnetic monopole is stable because of the nontrivial topology of the vacuum expectation value of the scalar field. Extending the discussion of 't Hooft [1], Julia-Zee [3] showed that a non-Abelian gauge theory with Higgs fields exhibits classical solution having both electrical and magnetic charges, which are called dyons. However the electric charge cannot exist without an accompanying magnetic charge and the stability arguments that apply to the dyonic solution. The conventional theories of monopoles and dyons show the presence of singularities. Dual potential approach has been explored to give singularity free formalism for the theories of monopoles and dyons. Cabibbo and Ferrari [4] gave a two potential theory for Abelian dyons. In this connection Zwanzinger [5, 6] developed the quantum field theory of particles with electric and magnetic charges as an extension of Schwinger's [7, 8] quantum field theory of particles with either electric or magnetic charge. Benjwal and Joshi [9] extended the Cabibbo-Ferrari [4] approach to the non-Abelian case by employing a non-Abelian field tensor. The same non-Abelian field tensor has also been used to obtain dyon solutions for temporal and non-temporal cases for a $SU(3) \otimes SU(3)$ gauge theory [10, 11].

Singleton [12] has given a (symmetric) formulation of electrodynamics which employs two four-vector potentials and the magnetic charge appears as a gauge charge without any singularities. In the present paper we extend Singleton's approach to a non-Abelian theory and derive dyon solutions for the temporal and non-temporal gauges. The distinguishing feature of the obtained solutions is the topological origin of both electric and magnetic charges.

Section II describes the Lagrangian and field equations. Particular finite energy temporal solutions for $SU(2)$ have been derived in Section III. The non-Abelian theory has been reduced to the Abelian Singleton theory in Section IV and thereby electrical and magnetic charges have been obtained. In Section V the mass of the obtained dyon solutions has been derived and non-temporal dyon solutions are presented in Section VI.

II. THE LAGRANGIAN AND FIELD EQUATIONS

The Lagrangian density that we consider is [10]

$$\mathcal{L} = -\frac{1}{4}A_{\mu\nu}^a A^{\mu\nu a} + \frac{1}{4}\tilde{B}_{\mu\nu}^a \tilde{B}^{\mu\nu a} + \frac{1}{2}(D_\mu^1 \phi_e^a)(D^{1\mu} \phi_e^a) + \frac{1}{2}(D_\mu^2 \phi_g^a)(D^{2\mu} \phi_g^a) + V(\phi_e^a, \phi_g^a) \quad (2.1)$$

where

$$\begin{aligned} A_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - ef^{abc}A_\mu^b A_\nu^c, & D_\mu^1 \phi_e^a &= \partial_\mu \phi_e^a - ef^{abc}A_\mu^b \phi_e^c, \\ B_{\mu\nu}^a &= \partial_\mu B_\nu^a - \partial_\nu B_\mu^a - gf^{abc}B_\mu^b B_\nu^c, & D_\mu^2 \phi_g^a &= \partial_\mu \phi_g^a - gf^{abc}B_\mu^b \phi_g^c, \\ \tilde{B}_{\mu\nu}^a &= \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}B^{\rho\sigma a}. \end{aligned} \quad (2.2)$$

The potential energy has the form $V(\phi_e^a, \phi_g^a) = -\eta(\phi_e^a \phi_e^a + \phi_g^a \phi_g^a - \xi^2)^2$ where η and ξ are real constants with $\eta \geq 0$. The gauge group under consideration is $SU(2)$, for which the structure constants $f^{abc} = \varepsilon^{abc}$. The Lagrangian density (2.1) is invariant under the following two independent sets of gauge transformations

$$\begin{aligned} \phi_e &\rightarrow U_1 \phi_e, & A_\mu &\rightarrow U_1 A_\mu U_1^{-1} + \frac{\iota}{e}(\partial_\mu U_1)U_1^{-1}, \\ \phi_g &\rightarrow U_2 \phi_g, & B_\mu &\rightarrow U_2 B_\mu U_2^{-1} + \frac{\iota}{g}(\partial_\mu U_2)U_2^{-1}, \end{aligned} \quad (2.3)$$

where,

$$\begin{aligned} \phi_e &= \phi_e^a T^a, & A_\mu &= A_\mu^a T^a, & U_1 &= \exp(-\iota \theta_1^a T^a), \\ \phi_g &= \phi_g^a T^a, & B_\mu &= B_\mu^a T^a, & U_2 &= \exp(-\iota \theta_2^a T^a), \end{aligned} \quad (2.4)$$

with T^a being the generators of the gauge group ($SU(2)$ for this case) and θ 's are some space-time parameters.

The Lagrangian density (2.1) gives the field equations

$$\begin{aligned} D_\mu^1 A^{\mu\nu a} - ef^{abc} \phi_e^b D^{1\nu} \phi_e^c &= 0, \\ D_\mu^2 B^{\mu\nu a} - gf^{abc} \phi_g^b D^{2\nu} \phi_g^c &= 0, \\ D_\mu^1 (D^{1\mu} \phi_e^a) + 4\eta \phi_e^a (\phi_e^b \phi_e^b + \phi_g^b \phi_g^b - \xi^2) &= 0, \\ D_\mu^2 (D^{2\mu} \phi_g^a) + 4\eta \phi_g^a (\phi_e^b \phi_e^b + \phi_g^b \phi_g^b - \xi^2) &= 0. \end{aligned} \quad (2.5)$$

We assume that all fields ($A_\mu^a, B_\mu^a, \phi_e^a, \phi_g^a$) are static i.e. time-independent. We also assume the temporal gauge condition for the fields A_μ^a and B_μ^a , i.e.

$$A_\mu^0(\mathbf{r}) = 0 = B_\mu^0(\mathbf{r}). \quad (2.6)$$

Under these assumptions the field equations (2.5) reduce to

$$\begin{aligned}
D_i^1 A^{ija} - e f^{abc} \phi_e^b D^{1j} \phi_e^c &= 0, \\
D_i^2 B^{ija} - g f^{abc} \phi_g^b D^{2j} \phi_g^c &= 0, \\
D_i^1 (D^{1i} \phi_e^a) + 4\eta \phi_e^a (\phi_e^b \phi_e^b + \phi_g^b \phi_g^b - \xi^2) &= 0, \\
D_i^2 (D^{2i} \phi_g^a) + 4\eta \phi_g^a (\phi_e^b \phi_e^b + \phi_g^b \phi_g^b - \xi^2) &= 0
\end{aligned} \tag{2.7}$$

which are equations of motion of static sourceless Euclidean Yang-Mills fields A_μ^a and B_μ^a .

III. FINITE ENERGY SOLUTIONS

We closely follow Rajaraman (Section 3.4) [13] to derive a particular finite energy solution for this system. The Hamiltonian density of the system is

$$\begin{aligned}
\mathcal{H} &= \frac{1}{4} A_{ij}^a A^{ija} - \frac{1}{4} (\tilde{B}_{0i}^a \tilde{B}^{0ia} + \tilde{B}_{i0}^a \tilde{B}^{i0a}) - \frac{1}{2} (D_i^1 \phi_e^a) (D^{1i} \phi_e^a) \\
&\quad - \frac{1}{2} (D_i^2 \phi_g^a) (D^{2i} \phi_g^a) + \eta (\phi_e^a \phi_e^a + \phi_g^a \phi_g^a - \xi^2)^2 \\
&= \frac{1}{4} (A_{ij}^a)^2 + \frac{1}{4} (B_{ij}^a)^2 + \frac{1}{2} (D_i^1 \phi_e^a)^2 \\
&\quad + \frac{1}{2} (D_i^2 \phi_g^a)^2 + \eta (\phi_e^a \phi_e^a + \phi_g^a \phi_g^a - \xi^2)^2.
\end{aligned} \tag{3.1}$$

The total energy of the system given by $E = \int d^3x \mathcal{H}$ is clearly ≥ 0 . The energy vanishes (minimum) for [13]

$$A_i^a = 0 = B_i^a, \quad D_i^1 \phi_e^a = 0 = D_i^2 \phi_g^a, \quad (\phi_e^a)^2 + (\phi_g^a)^2 = \xi^2. \tag{3.2}$$

The system will have a finite energy only if the fields achieve the energy vanishing conditions of Eq. (3.2) at spatial infinity sufficiently fast, i.e.

$$r^{\frac{3}{2}} D_i^1 \phi_e^a \rightarrow 0, \quad r^{\frac{3}{2}} D_i^2 \phi_g^a \rightarrow 0, \quad (\phi_e^a)^2 + (\phi_g^a)^2 \rightarrow \xi^2 \quad \text{as} \quad r \rightarrow \infty. \tag{3.3}$$

A particular finite energy solution can be obtained by employing the 't Hooft-Polyakov [1, 2] (temporal) ansatz for the fields

$$\begin{aligned}
\phi_e^a &= \delta_i^a \frac{x^i}{r} F_1(r), & A_i^a &= \varepsilon_{aij} \frac{x^j}{r} W_1(r), \\
\phi_g^a &= \delta_i^a \frac{x^i}{r} F_2(r), & B_i^a &= \varepsilon_{aij} \frac{x^j}{r} W_2(r).
\end{aligned} \tag{3.4}$$

The boundary conditions (3.3) for finite-energy configuration then imply [13]

$$W_1(r) \rightarrow \frac{1}{er}, \quad W_2(r) \rightarrow \frac{1}{gr}, \quad \{F_1(r)\}^2 + \{F_2(r)\}^2 \rightarrow \xi^2 \quad \text{as } r \rightarrow \infty. \quad (3.5)$$

Plugging the ansatz (3.4) into the field equations (2.7), we get for the $\eta = 0$ case

$$\begin{aligned} r^2 K_1'' &= K_1(K_1^2 + H_1^2 - 1), & r^2 H_1'' &= 2H_1 K_1^2, \\ r^2 K_2'' &= K_2(K_2^2 + H_2^2 - 1), & r^2 H_2'' &= 2H_2 K_2^2, \end{aligned} \quad (3.6)$$

where

$$\begin{aligned} K_1(r) &= 1 - erW_1(r), & H_1(r) &= erF_1(r), \\ K_2(r) &= 1 - grW_2(r), & H_2(r) &= grF_2(r), \end{aligned} \quad (3.7)$$

and K_1'' denotes $\frac{d^2}{dr^2} K_1(r)$ and so on.

The solution of Eqs. (3.6), satisfying the constraints of finite energy (3.5) are [13, 14]

$$\begin{aligned} K_1 &= \frac{erC}{\sinh(erC)}, & H_1 &= \frac{erC}{\tanh(erC)} - 1, \\ K_2 &= \frac{gr\sqrt{\xi^2 - C^2}}{\sinh(gr\sqrt{\xi^2 - C^2})}, & H_2 &= \frac{gr\sqrt{\xi^2 - C^2}}{\tanh(gr\sqrt{\xi^2 - C^2})} - 1, \end{aligned} \quad (3.8)$$

where C is some constant such that $0 \leq C \leq \xi$.

IV. TOPOLOGICAL ELECTRIC AND MAGNETIC CHARGES

Maxwell's equations for electrodynamics can be symmetrized by introducing magnetic charge and current

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho_e, & \nabla \times \mathbf{B} &= \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}_e, \\ \nabla \cdot \mathbf{B} &= \rho_m, & -\nabla \times \mathbf{E} &= \frac{\partial \mathbf{B}}{\partial t} + \mathbf{J}_m. \end{aligned} \quad (4.1)$$

The \mathbf{E} and \mathbf{B} field vectors are distinguished from one another under the spatial inversion or parity transformation ($\mathbf{r} \rightarrow -\mathbf{r}$), \mathbf{E} being a vector ($\mathbf{E} \rightarrow -\mathbf{E}$) and \mathbf{B} being a pseudovector ($\mathbf{B} \rightarrow \mathbf{B}$). These definitions and Maxwell's equations then imply that \mathbf{J}_e and \mathbf{J}_m are vector and pseudovector, respectively under parity while ρ_e and ρ_m are scalar and pseudoscalar, respectively under parity.

Singleton [12] has given a formulation of electrodynamics with electric and magnetic charges which employs two four-potentials. In this formalism the electric and magnetic fields are given by

$$E_i = F^{i0} - \tilde{G}^{i0}, \quad B_i = G^{i0} + \tilde{F}^{i0}, \quad (4.2)$$

where the field tensors and their duals are defined in terms of the two potentials A^μ and B^μ as

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, & G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ \tilde{F}^{\mu\nu} &= \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}, & \tilde{G}^{\mu\nu} &= \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}. \end{aligned} \quad (4.3)$$

Here $\varepsilon^{\mu\nu\rho\sigma}$ is the completely antisymmetric tensor in four dimensions with the choice $\varepsilon^{0123} = +1$.

Following 't Hooft [1], we define two gauge invariant field tensors

$$\begin{aligned} \mathcal{F}_{\mu\nu} &= \hat{\phi}_e^a A_{\mu\nu}^a - \frac{1}{e} \varepsilon^{abc} \hat{\phi}_e^a D_\mu^1 \hat{\phi}_e^b D_\nu^1 \hat{\phi}_e^c, \\ \mathcal{G}_{\mu\nu} &= \hat{\phi}_g^a B_{\mu\nu}^a - \frac{1}{g} \varepsilon^{abc} \hat{\phi}_g^a D_\mu^2 \hat{\phi}_g^b D_\nu^2 \hat{\phi}_g^c, \end{aligned} \quad (4.4)$$

where $\hat{\phi}_e^a = \phi_e^a / \sqrt{\phi_e^a \phi_e^a}$ and $\hat{\phi}_g^a = \phi_g^a / \sqrt{\phi_g^a \phi_g^a}$. We adopt the signature $(-, +, +, +)$ so that $\varepsilon^{123} = \varepsilon_{123} = +1$. For a gauge in which $\hat{\phi}_e^a$ is constant, say in the three-direction, the tensor $\mathcal{F}_{\mu\nu}$ reduces to [1, 15] $\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3$, which is of the form of $F_{\mu\nu}$ (See Eq. (4.3)). Similarly in a gauge with constant $\hat{\phi}_g^a$, $\mathcal{G}_{\mu\nu}$ becomes similar to $G_{\mu\nu}$. Thus the 't Hooft tensors (4.4) reduce to the Singleton tensors (4.3) for particular gauge choices.

We now employ $\mathcal{F}_{\mu\nu}$ and $\mathcal{G}_{\mu\nu}$ instead of $F_{\mu\nu}$ and $G_{\mu\nu}$ in Eq. (4.2) to derive the electric and magnetic fields

$$E_i = \mathcal{F}^{i0} - \tilde{\mathcal{G}}^{i0}, \quad B_i = \mathcal{G}^{i0} + \tilde{\mathcal{F}}^{i0}. \quad (4.5)$$

As $\mathcal{F}^{i0} = 0 = \mathcal{G}^{i0}$ due to static and temporal-gauge choice for the fields we have

$$\begin{aligned} E_i &= -\tilde{\mathcal{G}}^{i0} = -\frac{1}{2} \varepsilon^{i0\rho\sigma} \mathcal{G}_{\rho\sigma} = \frac{1}{2} \varepsilon^{ijk} \mathcal{G}_{jk}, \\ B_i &= \tilde{\mathcal{F}}^{i0} = \frac{1}{2} \varepsilon^{i0\rho\sigma} \mathcal{F}_{\rho\sigma} = -\frac{1}{2} \varepsilon^{ijk} \mathcal{F}_{jk}. \end{aligned} \quad (4.6)$$

We also define two currents [16]

$$\begin{aligned} (k_e)_\mu &= \frac{1}{8\pi} \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{abc} \partial^\nu \hat{\phi}_e^a \partial^\rho \hat{\phi}_e^b \partial^\sigma \hat{\phi}_e^c, \\ (k_g)_\mu &= \frac{1}{8\pi} \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{abc} \partial^\nu \hat{\phi}_g^a \partial^\rho \hat{\phi}_g^b \partial^\sigma \hat{\phi}_g^c. \end{aligned} \quad (4.7)$$

Both these currents are conserved by construction itself i.e. $\partial^\mu (k_e)_\mu = 0 = \partial^\mu (k_g)_\mu$. The corresponding conserved (topological) charges are [16]

$$Q_1 = \int d^3x (k_e)^0, \quad Q_2 = \int d^3x (k_g)^0. \quad (4.8)$$

These charges can be interpreted as the topological winding numbers of the corresponding ϕ -fields. The currents (4.7) are related to the 't Hooft tensors (4.4) as

$$(k_e)_\mu = \frac{e}{8\pi} \varepsilon_{\mu\nu\rho\sigma} \partial^\nu \mathcal{F}^{\rho\sigma}, \quad (k_g)_\mu = \frac{g}{8\pi} \varepsilon_{\mu\nu\rho\sigma} \partial^\nu \mathcal{G}^{\rho\sigma}. \quad (4.9)$$

The magnetic flux can be computed using Eqs. (4.6), (4.7) and (4.9) as

$$\nabla \cdot \mathbf{B} = \partial^i B_i = \partial^i \left(-\frac{1}{2} \varepsilon^{ijk} \mathcal{F}_{jk} \right) = \frac{1}{2} \varepsilon_{0\nu\rho\sigma} (\partial^\nu \mathcal{F}^{\rho\sigma}) = -\frac{4\pi}{e} (k_e)^0. \quad (4.10)$$

The magnetic charge given by the volume integral of the magnetic flux is

$$q_m = \int d^3x (\nabla \cdot \mathbf{B}) = \int d^3x \left(-\frac{4\pi}{e} \right) (k_e)^0 = -\frac{4\pi}{e} Q_1, \quad (4.11)$$

where Q_1 is the topological charge (See Eq. (4.8)). Similarly the electric charge is

$$q_e = \frac{4\pi}{g} Q_2. \quad (4.12)$$

Both these charges, derived for the temporal gauge choice, are topological in origin. The particular solutions (3.8), derived above by employing the ansatz (3.4), correspond to $Q_1 = 1 = Q_2$ and thus are dyon-solutions with electric and magnetic charges $4\pi/g$ and $-4\pi/e$ respectively.

V. MASS OF THE DYON

A Bogomol'nyi [17] type bound can be derived for the dyon-solutions derived above. The energy of the system (3.1) for the $\eta = 0$ case can be written as

$$E = \int d^3x \left[\sum_{i,j,a} \left\{ \frac{1}{4} (A_{ij}^a - \varepsilon_{ijk} D_k^1 \phi_e^a)^2 + \frac{1}{4} (B_{ij}^a - \varepsilon_{ijk} D_k^2 \phi_g^a)^2 \right\} + \frac{1}{2} \varepsilon_{ijk} A_{ij}^a D_k^1 \phi_e^a + \frac{1}{2} \varepsilon_{ijk} B_{ij}^a D_k^2 \phi_g^a \right]. \quad (5.1)$$

The third and fourth terms in the above integral (5.1) can be written as surface integrals of the form

$$\oint d\sigma_k \left(\frac{1}{2} \varepsilon_{kij} A_{ij}^a \phi_e^a \right) \quad \text{and} \quad \oint d\sigma_k \left(\frac{1}{2} \varepsilon_{kij} B_{ij}^a \phi_g^a \right), \quad (5.2)$$

where the integration is to be performed at the surface of a two-sphere at spatial infinity. At spatial infinity, $D_i^1 \phi_e^a = 0$, due to boundary condition for finite energy (see Eq. (3.3)). Also, $\hat{\phi}_e^a = \phi_e^a/C$ at spatial infinity (Using Eqs. (3.4), (3.7), (3.8)), so that the space-space component of the 't Hooft tensor $\mathcal{F}_{\mu\nu}$ (4.4) is $\mathcal{F}_{ij} = (\phi_e^a/C)A_{ij}^a$. Hence, the magnetic field (4.6)(at spatial infinity) can be written as $B_i = -\frac{1}{2}\varepsilon^{ijk}(\phi_e^a/C)A_{jk}^a$. The first surface integral in Eq. (5.2) can therefore be rewritten as

$$\oint d\sigma_k \left(\frac{1}{2} \varepsilon_{kij} A_{ij}^a \phi_e^a \right) = \oint d\sigma_k (-CB^k) = -C(q_m) = \frac{4\pi}{e}C, \quad (5.3)$$

where use of Eq. (4.11), with $Q_1 = 1$, is implied. The second surface integral in Eq. (5.2), similarly, simplifies to

$$\oint d\sigma_k \left(\frac{1}{2} \varepsilon_{kij} B_{ij}^a \phi_g^a \right) = \frac{4\pi}{g} \sqrt{\xi^2 - C^2}. \quad (5.4)$$

Using Eqs. (5.3) and (5.4), Eq. (5.1) becomes

$$\begin{aligned} E &= \frac{4\pi}{e}C + \frac{4\pi}{g} \sqrt{\xi^2 - C^2} \\ &\quad + \int d^3x \sum_{i,j,a} \left\{ \frac{1}{4} (A_{ij}^a - \varepsilon_{ijk} D_k^1 \phi_e^a)^2 + \frac{1}{4} (B_{ij}^a - \varepsilon_{ijk} D_k^2 \phi_g^a)^2 \right\} \\ &\geq \frac{4\pi}{e}C + \frac{4\pi}{g} \sqrt{\xi^2 - C^2}. \end{aligned} \quad (5.5)$$

The energy reaches its minimum when [17]

$$A_{ij}^a = \varepsilon_{ijk} D_k^1 \phi_e^a, \quad B_{ij}^a = \varepsilon_{ijk} D_k^2 \phi_g^a. \quad (5.6)$$

These are the Bogomol'nyi-type conditions for the topological dyon. The equality in Eq. (5.5) corresponds to the mass of the dyon solution (3.8). The energy of the monopole configuration in the BPS limit is independent of the properties of the gauge fields and completely defined by the Higgs field alone.

VI. NON-TEMPORAL SOLUTIONS

If instead of the temporal gauge choice (2.6) we use the Julia-Zee ansatz [3] for A_0^a and B_0^a i.e.

$$A_0^a = \frac{x^a}{er^2} J_1(r), \quad B_0^a = \frac{x^a}{gr^2} J_2(r) \quad (6.1)$$

along with those in Eq. (3.4), the solutions (3.8) are modified as [3]

$$\begin{aligned}
K_1 &= \frac{erC}{\sinh(erC)}, & K_2 &= \frac{gr\sqrt{\xi^2 - C^2}}{\sinh(gr\sqrt{\xi^2 - C^2})}, \\
H_1 &= \cosh \gamma_1 \left[\frac{erC}{\tanh(erC)} - 1 \right], & H_2 &= \cosh \gamma_2 \left[\frac{gr\sqrt{\xi^2 - C^2}}{\tanh(gr\sqrt{\xi^2 - C^2})} - 1 \right], \\
J_1 &= \sinh \gamma_1 \left[\frac{erC}{\tanh(erC)} - 1 \right], & J_2 &= \sinh \gamma_2 \left[\frac{gr\sqrt{\xi^2 - C^2}}{\tanh(gr\sqrt{\xi^2 - C^2})} - 1 \right], \quad (6.2)
\end{aligned}$$

where γ_1 and γ_2 are arbitrary constants. For the temporal case \mathcal{F}^{i0} and \mathcal{G}^{i0} were zero and the electric and magnetic charges were as given by Eq. (4.6). For the non-temporal case, \mathcal{F}^{i0} and \mathcal{G}^{i0} being non-zero, the electric and magnetic fields are to be computed using Eq. (4.5). The magnetic charge calculated in Eq. (4.11) now contains an extra contribution $\int d^3x (\partial^i \mathcal{G}^{i0})$ which simplifies to $(1/g) \int d^3x (J_2''/r)$ on using Eqs. (2.2), (3.4), (4.4) and (6.1). This integral when evaluated using Eq. (6.2) gives $(4\pi/g) \sinh \gamma_2$ [14]. Therefore, the total magnetic charge of the non-temporal solution (6.2) is

$$q_m = -\frac{4\pi}{e} + \frac{4\pi}{g} \sinh \gamma_2. \quad (6.3)$$

Similarly, the total electric charge of the solution (6.2) is

$$q_e = \frac{4\pi}{g} + \frac{4\pi}{e} \sinh \gamma_1. \quad (6.4)$$

Thus, the non-temporal solutions (6.2) are also dyonic solutions. There is a possibility of the charge q_e (or q_m), of the non-temporal solution (6.2), being zero for particular value of γ_2 (or γ_1), so that we are left with a pure monopole (or pure electric charge) only.

VII. SUMMARY

The two-potential Cabibbo-Ferrari Abelian theory of magnetic monopole has been extended by a non-Abelian theory. Using the t Hooft-Polyakov ansatz, finite energy solutions have been obtained in the temporal gauge. These solutions carry (topological) electric and magnetic charges and thus are dyonic-solutions. The energy of these solutions is found to be bounded from below. The temporal gauge choice when extended to the non-temporal gauge (using the Julia-Zee ansatz) again yields solutions carrying electric and magnetic charges.

In the conventional (dual) theories of magnetic monopoles and dyons the magnetic charge (electric charge) is topological in origin. The distinguishing feature of the solutions obtained

here is that both the charges (electric and magnetic) are completely topological in origin, for the temporal case and partially-topological for the non-temporal case. Furthermore, in the conventional theories the dyon solutions emerge only in the non-temporal case whereas in this two-potential formulation the dyon solutions are present even in the temporal case. The energy of the temporal case dyon solutions as derived in Section V is positive definite and bounded from below which guarantees the stability of the obtained solutions. However, a two-potential theory implies presence of two photons when the theory is quantized. Experimentally, only one photon is observed in nature. A possible remedy for this might be (a large) mass generation for one of the photons via symmetry breaking mechanism [12, 18]. It is well known that monopoles and dyons emerge as essential ingredients in the dual superconductor models of QCD in context with the quark confinement problem [19–22]. It would be interesting to investigate the issue of confinement through the condensation of these topological dyons.

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